TURBULENT BOUNDARY LAYER AT THE INITIAL PORTION OF A PIPE WITH ROUGH WALLS

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The development of a turbulent boundary layer at the initial portion of a pipe with rough walls is considered in the framework of the boundary-layer theory. It is shown that the consideration of roughness can be carried out by introducing into the "standard" law of friction a function which takes into account this factor. An experimental investigation is carried out on a test portion of a pipe with natural roughness whose relative value equals 10^{-3}. The range of Reynolds numbers is $5.1 \cdot 10^4$–$3.4 \cdot 10^5$. The method of calculation proposed here leads to results which satisfactorily agree with the data of the experimental investigation.

The system of equations for the problem being considered is represented in the form [1]

$$\frac{dR^{**}}{dX} + (1+H)\frac{R^{**}}{W_0}\frac{dW_0}{dX} = R\frac{c_f}{2}, \qquad 4HR^{**} = R - R_1 \tag{1}$$

Here and in what follows the notation is that generally adopted [1].

To determine the friction, we use the relationship [1]

$$\frac{c_f}{2} = \left[\int_{\omega_1}^{1} \sqrt{\frac{\rho}{\rho_0}\frac{\tau_0}{\tau}\frac{\tau_w}{\tau_{0w}}}\, d\omega\right]^2 \left[\int_{\xi_1}^{1} \frac{d\xi}{\varkappa\xi}\right]^{-2} \tag{2}$$

For isothermal flow of an incompressible liquid the integration of (2) leads to the following result:

$$\frac{c_f}{2} = \left(\frac{1-\omega_1}{\varkappa^{-1}\ln \xi_1}\right)^2 \tag{3}$$

The velocity on the boundary of a laminar sublayer in the general case can be expressed by the relationship [2]

$$\omega_1 = \eta_1 \sqrt{c_f/2} \tag{4}$$

where the factor of proportionality η_1 for otherwise identical conditions is a function of state of the surface. The absence of experimental data on friction for a turbulent motion of a liquid in the initial portion of the pipe does not allow us to exactly determine its value. However, if we assume that the effect of the state of the surface both in the region of stabilized flow and in the portion of hydrodynamic stabilization is single-valued, then η_1 can be found by the following method.

The coefficient of resistance during a motion of a liquid over the basic portion of the pipe is given by the relationship [3]

$$-\frac{dp}{dX} = \lambda \frac{\rho w_{01}^2}{2} \tag{5}$$

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The expression (5) is written with the assumption that the velocity profile at the input is uniform.

On the other hand, from the equilibrium of an elemental volume of the liquid we have

$$-\frac{dp}{dX}=4\tau_w, \qquad \tau_w=\frac{c_f}{2}\rho_0 w_0^2 \tag{6}$$

Simultaneously solving (4) and (6), we obtain the connection between the coefficients of friction and resistance over the portion of stabilized flow.

$$\lambda=8\frac{c_f}{2}\left(\frac{w_{0X}}{w_{01}}\right)^2 \tag{7}$$

The second equation (1) is conveniently represented in the form

$$\frac{w_0}{w_{01}}=\left(1-2\frac{\delta^*}{r_0}\right)^{-1} \tag{8}$$

The thickness of displacement entering (8), in the general case is expressed by the relationship

$$\delta^*=\int_0^\delta(1-\omega)\left(1-\frac{y}{r_0}\right)dy, \omega=1+\frac{1}{\varkappa}\sqrt{\frac{c_f}{2}}\ln\zeta \tag{9}$$

Integrating (9), we have

$$\frac{\delta^*}{\delta}=\frac{1}{\varkappa}\sqrt{\frac{c_f}{2}}\left(1-\frac{1}{4}\cdot\frac{\delta}{r_0}\right) \tag{10}$$

Simultaneously solving (7), (8), and (10), we find that

$$\frac{c_f}{2}=\left(\frac{3}{2\varkappa}+\sqrt{\frac{8}{\lambda}}\right)^{-2}, \quad \frac{w_{0X}}{w_{01}}=\sqrt{\frac{\lambda}{4c_f}}, \quad R_X=R_1W_{0X} \tag{11}$$

In [2] it is shown that the thickness of the laminar sublayer can be expressed by the relationship

$$\zeta_1=\eta_1\left(\frac{R}{2}\frac{\delta}{r_0}\right)^{-1}\left(\frac{c_f}{2}\right)^{-1/2} \tag{12}$$

Substituting (4) and (12) into (3), we obtain

$$\eta_1\varkappa-\ln\eta_1=\frac{\varkappa}{\sqrt{c_f/2}}-\ln\left(\sqrt{\frac{c_f}{2}}\frac{R_1}{2}\right) \tag{13}$$

Using the experimental relationship between λ and R_1 and putting $\delta = r_0$, we can find the quantity η_1.

In Fig. 1 dots represent the results of processing the test data of [4], obtained for various values of the relative roughness k_a/r_0: 1 – 0, 0.125; 2 – 0.004; 3 – 0.0166; 4 – 0.008; 5 – 0.001. The dots are located on a curve whose equation is given by the relationship

$$\eta_1=8+3.6\left(\frac{\zeta_1-K}{\zeta_1}\right)^{5/2}, \quad \zeta_1-K\geqslant 0, \quad K=\frac{k_a}{\delta} \tag{14}$$

The system of equations (3), (4), (12), and (14) completely defines the friction coefficient as a function of the Reynolds number.

In Fig. 2 solid lines are used to represent the results of calculating the velocity profile from the expressions (3) and (9) for k_a/r_0: 1 – 0.0333; 2 – 0; 3 – 0.00795; 4 – 0.0198. Here, by dots, for the same values of k_a/r_0, we have marked the test data of [3].

Just as experiment, so also the calculation indicates a depression of the velocity profile in the turbulent core of the boundary layer as the relative roughness increases.

Substituting ω from (9) into the expression for the thickness of the impulse loss

$$\delta^{**}=\int_0^\delta\omega(1-\omega)\left(1-\frac{y}{r_0}\right)dy \tag{15}$$

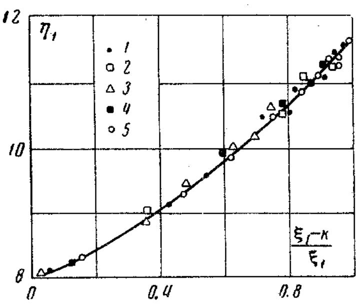

Fig. 1

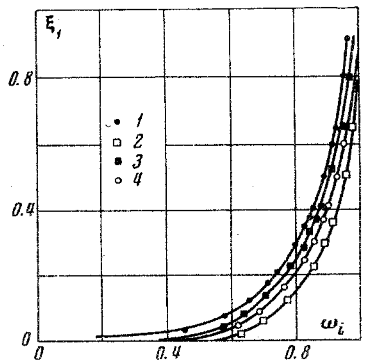

Fig. 2

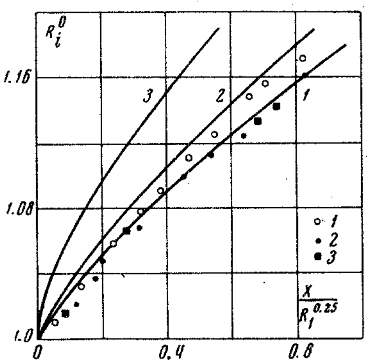

Fig. 3

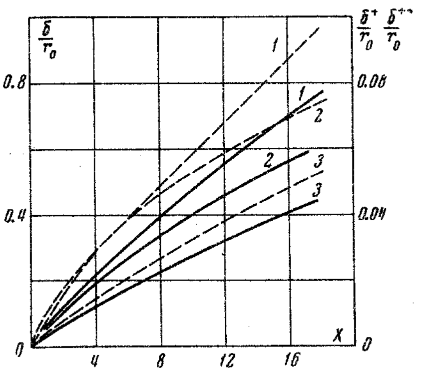

Fig. 4

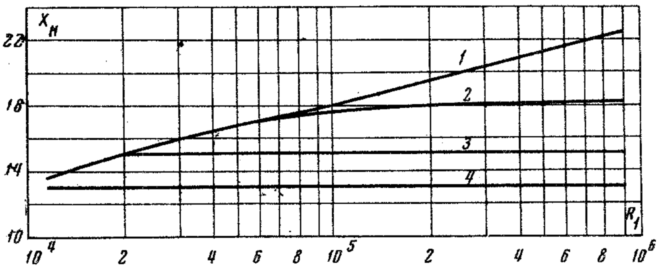

Fig. 5

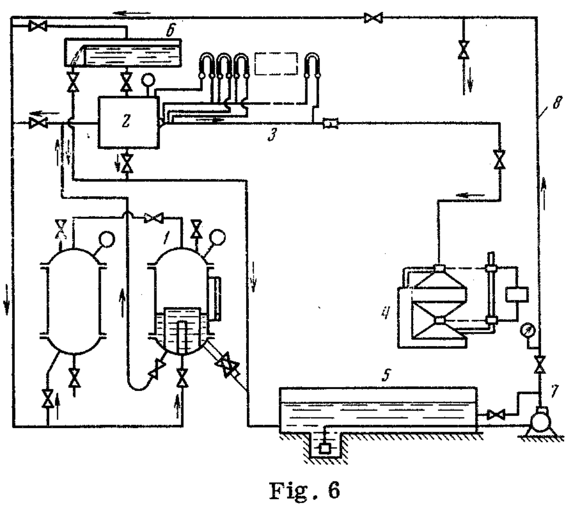

Fig. 6

we obtain

$$\frac{\delta^{**}}{\delta}=\frac{\delta^{*}}{\delta}-\frac{c_f}{\varkappa^2}\left(1-\frac{1}{8}\frac{\delta}{r_0}\right) \tag{16}$$

Dividing (16) by (10), for the shape parameter H we obtain

$$H=\left[1-\frac{1}{\varkappa}\sqrt{\frac{c_f}{2}}\left\{1+2\left(1+\sqrt{1-2\varkappa\sqrt{\frac{2}{c_f}}\frac{R-R_1}{4R}}\right)^{-1}\right\}\right]^{-1} \tag{17}$$

An analysis of (9), (10), (16), and (17) shows that all integral characteristics are functions of the thickness of the boundary layer.

Introducing the Reynolds criterion into (10) and solving the latter for δ/r_0, with (1) taken into account, we have

$$\delta/r_0=2\left(1-\sqrt{1-2\varkappa\sqrt{2/c_f}\,\frac{R-R_1}{4R}}\right) \tag{18}$$

Thus, the system (1), (3), (4), (12), (14), (18) completely determines the development of a dynamic boundary layer in the initial portion of the pipe with rough walls.

For its solution Eq. (1) is transferred into

$$\frac{dR}{dX}=4RH\frac{c_f}{2}\left[1-a\frac{R-R_1}{H}+(1+H)\frac{R-R_1}{H}\right]^{-1} \tag{19}$$

Equation (19) was solved by the Runge–Kutta method on a digital computer. Here the coefficient a was calculated from (17).

In Fig. 3 the solid lines were used to represent the results of the calculation of the distribution of the Reynolds number R along the length of the initial portion for the values of relative roughness k_a/r_0: 1 – 0.2; 2 – 10^{-3}; 3 – 10^{-2}; and $R_1 = 2 \cdot 10^5$. Here dots show the experimental data obtained in [5,6] for $k_a/r_0 = 0$, and the data by the author for $k_a/r_0 = 10^{-3}$. We see that the roughness only slightly influences the relative value of the Reynolds number R, plotted against the velocity on the axis of the pipe.

In Fig. 4 the dashed lines 1-3 show: 1 – δ/r_0; 2 – δ^*/r_0; 3 – δ^{**}/r_0; here $R_1 = 10^6$, $k_a/r_0 = 10^{-3}$. The solid lines show the same quantities for $k_a/r_0 = 0$. We observe an increase in the integral characteristics of the boundary value, as the roughness increases.

In Fig. 5 we have presented the results of the calculation of the length of the initial portion, dependent on the Reynolds number R_1, and the relative roughness. The following values correspond to the lines 1-4: 1 – $k_a/r_0 = 0$; 2 – 10^{-3}; 3 – $5 \cdot 10^{-3}$; 4 – 10^{-2}. The length of the portion of hydrodynamic stabilization decreases as the ratio k_a/r_0 increases.

This is explained by the growth in the shear stresses which take place on the surface next to the stream, and hence by the deeper penetration of their influence into the stream. This gives rise to a depression of the velocity profile across the boundary layer, and accelerates the growth of the latter.

An experimental investigation of the problem being considered here was carried out on a hydrodynamic installation schematically represented in Fig. 6. Its basic elements are: 1) towerless pressure system; 2) forechamber; 3) test section; 4) measuring system; 5) overflow reservoir; 6) head tank; 7) pump; 8) connecting pipe lines with locking devices.

The towerless pressure system ensured a steady feed (without pulsations) of water to the forechamber, a smooth regulation of the value of flow and allowed it to be maintained constant during a test.

The forechamber is made in the form of a cylinder having a diameter of 1.2 m. An input device maintaining a smooth flow into the test section is mounted on to its front cover. A cylindrical pipe of stainless steel with an inside diameter of 99.8 mm and a length of 6500 mm was used as the test section.

Over its entire length the test section was divided into 24 segments with the coordinates X = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65. In each of them, diametrically opposite on the horizontal center line two holes of 1.2-mm diameter were drilled for the sampling of the static pressure. After drilling burrs were carefully removed from the inner edges of the holes, with the aim of eliminating incorrect readings. Outside each two samplers belonging to the same section were connected with one another by an aluminum tube, of $D_y = 4$ mm, through a special fitting, to average the quantity being measured. In addition, at the segments with coordinates X = 2, 6, 12, 20, 65 there were holes with a diameter of 3.5 mm at the top for introducing total head tubes into the stream, while on the outside surface of the pipe at these points there were devices for fixing special coordinators which served to displace the tubes within the limits from 0 to 60 mm with a minimum step of 0.05 mm.

The total head tube was used to take the velocity profile at chosen cross segments of the test section. Its receiver orifice was elliptical 1.2×0.8 mm^2.

The registration of pressure in all cases was effected by inverted U-shaped piezometers.

The flow of water was measured by the volume measuring system 4 provided with a device for automatic counting of fillings of a calibrated volume. The accuracy of flow measurement from the data of the tests carried out amounts to $\pm 0.2\%$. The volume of the overflow reservoir 5 equals 14 m^3. This ensures continuous operation of the pump while the measuring system is being filled.

The value of the relative roughness was determined from the measured coefficient of resistance λ, expressed by the relationship (5), and from the well-known Nikuradze expression [3]. It was found to be equal to $k_a/r_0 = 10^{-3}$. The tests were carried out under steady-state conditions. During the time of the experiment the following quantities were measured: the pressure and temperature of the water in the forechamber, the distribution of the static pressure along the length of the test section, the distribution of the total pressure across the section of the pipe line, and the volume throughput.

The results of the measurements were processed according to the following method.

We know [6] that the shear stress on the wall during the motion of the liquid in the pipe line can be expressed by the relationship

$$\tau_w = -\frac{1}{4}\left(\frac{d}{dX}\frac{p}{\rho_{01}w_{01}^2} + \frac{dM}{dX}\right), \quad M = \int_0^1 \left(\frac{\rho w}{\rho_{01}w_{01}}\right)^2 d\left(\frac{r}{r_0}\right)^2 \tag{20}$$

where M is the momentum of the liquid flowing through any section of the pipe line. The quantity $\rho_{01}w_{01}^2 \cdot$ (dM/dX) can be represented in the form

$$\rho_{01}w_{01}^2\frac{dM}{dX} = \frac{d}{dX}\left[w_0\left(\rho_{01}w_{01} - \frac{\rho_0 w_0 - \rho_{01}w_{01}}{H}\right)\right] \tag{21}$$

Noting that

$$\frac{\rho_0 w_{0i}^2}{2} = p^* - p_i = \Delta h_i \tag{22}$$

from (21), as a result of differentiation, we obtain

$$\begin{aligned}\rho_{01}w_{01}^2\frac{dM}{dX} = &\left(\sqrt{\frac{\Delta h_1}{\Delta h_i}} - \frac{2}{H} + \frac{1}{H}\sqrt{\frac{\Delta h_1}{\Delta h_i}}\right)\frac{d\Delta h_i}{dX} \\ &+ (2\Delta h_i - 2\sqrt{\Delta h_i \Delta h_1})\frac{1}{H^2}\frac{dH}{dX}\end{aligned} \tag{23}$$

Substituting (23) into (20), we obtain the working expression for the calculation of the coefficient of friction from the quantities measured experimentally.

$$\begin{aligned}\frac{c_f}{2} = \frac{1}{8\Delta h_i}\Bigg\{&\frac{d\Delta h_i}{dX} - \left(\sqrt{\frac{\Delta h_1}{\Delta h_i}} - \frac{2}{H_i} + \frac{1}{H_i}\sqrt{\frac{\Delta h_1}{\Delta h_i}}\right)\frac{d\Delta h_i}{dX} \\ &- (2\Delta h_i - 2\sqrt{\Delta h_i \Delta h_1})\frac{1}{H_i^2}\frac{dH_i}{dX}\end{aligned} \tag{24}$$

The calculation according to the expression (21) was carried out in the following sequence. At the beginning, in the first approximation, it was assumed that dH/dX = 0. By the method of least squares for a number of points we found a relationship of the form

$$\Delta h_i = a + bX + cX^2 \tag{25}$$

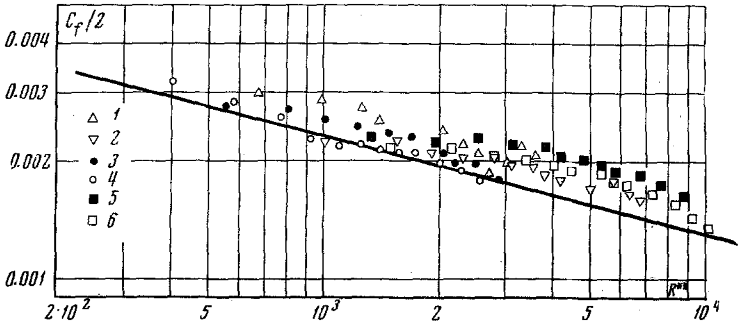

Fig. 7

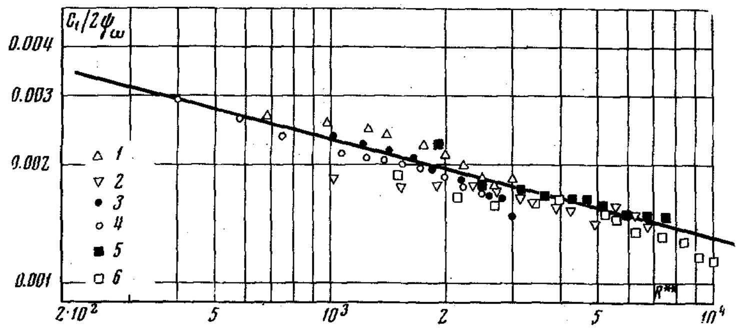

Fig. 8

which was differentiated, and the resulting expression was substituted into (24). For all these points we determined the coefficients of friction, and from the expression (17) we calculated the distribution of the shape parameter H_i. The value of H_i thus found was then approximated by a relationship of the form (25), with the difference that instead of Δh_i the quantity H was used.

The expression (25) was differentiated, and the gradient of the shape parameter was substituted into the expression (24). In this way the value of the coefficient of friction was calculated in the second approximation. All operations were repeated as long as the relationship

$$|(c_{f_i} - c_{f_{i-1}})/c_{f_i}| \leqslant 10^{-4} \tag{26}$$

was not fulfilled at each point.

The values of the coefficients of friction thus found were subsequently used to calculate the Reynolds number, plotted against the thickness of the loss of impulse. The calculated relationship for R** can be obtained from the momentum equation via simple transformations. We have

$$R_i^{**} = \exp\ln\sqrt{\frac{\Delta h_1}{\Delta h_i}} \int\limits_0^{X_i} \sqrt{2\rho}\,\frac{D}{\mu}\sqrt{\Delta h_i}\,\frac{c_{fi}}{2}\left(\frac{\sqrt{\Delta h_i}+\sqrt{\Delta h_1}}{2\sqrt{\Delta h_i}}\right)\exp\ln\sqrt{\frac{\Delta h_i}{\Delta h_1}}\,dX \tag{27}$$

In Fig. 7 we have presented the results for such processing for R_1: 1 – 10.2 · 10^4; 2 – 20.7 · 10^4; 3 – 8.9 · 10^4; 4 – 7.4 · 10^4; 5 – 26.7 · 10^4; 6 – 31 · 10^4. Here we also have represented by the line the relationship

$$\frac{c_{f0}}{2} = \frac{0.0128}{R^{**0.25}} \tag{28}$$

which according to the data of the publications [1, 3-7] well agrees with the test data obtained for a flow along smooth surfaces. We see that the test data lie above this relationship.

In view of the fact that the walls of the pipe were rough, it is natural to assume that the difference existing between the results thus obtained and the relationship (28) is explained by the presence of the roughness. Therefore, an attempt was made to take into account this effect, using the relationships obtained earlier.

We denote by Ψ^* the value of the coefficient of friction

$$\Psi^* = (c_f / c_{f0})_{R^{**}} \tag{29}$$

The relationship (29) can be obtained from (3) by substituting into it the values of the velocity on the boundary of the laminar sublayer and its thickness calculated for the same number R^{**} for smooth and rough surfaces. We have

$$\Psi^*_{R^{**}} = \left(\frac{\varkappa\eta_{10} - \ln\zeta_{10}}{\varkappa\eta_1 - \ln\zeta_1}\right)^2 \tag{30}$$

$$\xi_1 = \eta_1\left(R^{**}\frac{r_0}{\delta^{**}}\right)^{-1}\left(\frac{c_{f0}}{2}\Psi^*\right)^{-1/2}$$

where η_1 and $\frac{1}{2}c_{f_0}$ are given respectively by the expressions (13) and (28).

In Fig. 8 the test data on friction in the initial portion of the pipe is processed, with the relationship (30) taken into account. We see that consideration of the effect of roughness leads to an agreement between the test data and the relationship (28) which is valid for flow round a smooth impermeable plate. But this allows us to draw the conclusion that the law of friction for flow along rough surfaces can be written in the form

$$\frac{c_f}{2} = \frac{0.0128\Psi^*}{R^{**0.25}}$$

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